

EE203001 Linear Algebra
Quiz #5 03/25/2003

Quiz Problems:

1. (10 pts.) In the real linear space $C(-1, 1)$, let $(f, g) = \int_{-1}^1 f(t)g(t)dt$. Consider the three functions $u_1(t) = 1, u_2(t) = t, u_3(t) = 1 + t$. Prove that two of them are orthogonal, two make an angle $\pi/3$ with each other, and two make an angle $\pi/6$ with each other. (Note that $\cos \theta = \frac{(x,y)}{\|x\|\|y\|}$.)
2. In the real linear space $C(1, e)$, define an inner product by the equation $(f, g) = \int_1^e (\log x)f(x)g(x)dx$.
 - (a) (5 pts.) If $f(x) = \sqrt{x}$, compute $\|f\|^2$.
 - (b) (5 pts.) If $f(x) = 1$, compute $\|f\|^2$.
 - (c) (5 pts.) By (a) and (b), find a linear polynomial $g(x) = a + bx$ orthogonal to the constant function $f(x) = 1$.
3. Let V consist of all sequence $\{x_n\}$ of real numbers for which the series $\sum_{n=1}^{\infty} x_n^2$ converges. If $x = \{x_n\}$ and $y = \{y_n\}$ are two elements of V , define $(x, y) = \sum_{n=1}^{\infty} x_n y_n$.
 - (a) (5 pts.) Prove that this series converges absolutely, i.e the series $\sum_{n=1}^{\infty} |x_n y_n|$ converges.
 - (b) (5 pts.) Compute (x, y) if $x_n = 1/n$ and $y_n = 1/(n+2)$ for $n \geq 1$.
 - (c) (5 pts.) Compute (x, y) if $x_n = 3^{-n}$ and $y_n = 1/n!$ for $n \geq 1$.