# EE203001 Linear Algebra
# Quiz #4    03/17/2003

**Quiz Problems:**

1. (10 pts.) In the linear space of all real polynomials, the subspace spanned by $\{1, t, t^2\}$ can be described as the subspace of all polynomials of degree $\leq 2$. Please describe the subspace spanned by $\{1+t, (1+t)^2, (1+t)^3\}$ similarly, and determine the dimension of the subspace.

2. (20 pts.) In the linear space $P_1$ of all real polynomials of degree $\leq 1$, define

$$(f, g) = f(0)g(0) + f(1)g(1).$$

   (a) Prove that $(f, g)$ is an inner product for $P_1$.

   (b) Compute $(f, g)$ when $f(t) = t$ and $g(t) = at + b$.

   (c) If $f(t) = t$, find all linear polynomials $g$ such that $(f, g) = 0$. Denote the set of all such $g$'s as $f^\perp$, i.e. $f^\perp = \{g \in P_1 | (f, g) = 0\}$. Is $f^\perp$ a subspace of $P_1$? If $f^\perp$ is a subspace of $P_1$, compute its dimension.

3. (10 pts.) Let $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n)$ be vectors in $\mathbb{R}^n$, determine whether $(x, y)$ is an inner product for $\mathbb{R}^n$ if $(x, y)$ is defined by the formula given. In case $(x, y)$ is not an inner product, tell which axioms are not satisfied.

   (a). $(x, y) = \sum_{i=1}^n (x_i + y_i)^2 - \sum_{i=1}^n x_i^2 - \sum_{i=1}^n y_i^2$.

   (b). $(x, y) = \sum_{i=1}^n (x_i + y_i)^2 - \sum_{i=1}^n x_i y_i$.