EE203001 Linear Algebra
Quiz #3   03/11/2003

Definitions:

1. Injective (one-to-one): A function $f : X \to Y$ is said to be injective if $f(x) = f(x')$ for two elements x, x' in X, then $x = x'$.

2. $g \circ f$: The composite $g \circ f$ of two functions $f : X \to Y$ and $g : Y \to Z$ is the function from X to Z defined as $(g \circ f)(x) = g(f(x))$, $\forall x \in X$.

3. 1_X(the identity function on X): $1_X(x) = x$, $\forall x \in X$.

Quiz Problems:

1. (10%) Let $f : X \to Y$ be a function. Please prove that f is injective if and only if there is a function $g : Y \to X$ such that $g \circ f = 1_X$. (Hint: Choose $g(y) = x$, if $y \in f(X)$ and $f(x) = y$, or $g(y) = x_0$, for an arbitrarily selected $x_0 \in X$, if $y \notin f(X)$.)

2. (10%) Prove that the space of all polynomials cannot be spanned by any finite set of polynomials.

3. Let V be the linear space of all real-valued functions on the real line. Determine whether each of the following subsets of V is linearly dependent or independent.

   (a) (10%) $\{\sin x, \sin 2x\}$.
   (b) (10%) $\{1, e^{ax}, e^{bx}\}$, $a \neq b$.

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