EE203001 Linear Algebra

Solutions to Quiz #3 Spring Semester, 2003

1. If f is injective, then for each y in the image $f(X)$ of X under f there is a unique x in X with $f(x) = y$. Choose an arbitrary fixed x_0 in X and define a function $g : Y \to X$ as

$$g(y) = \begin{cases} x & \text{, if } y \in f(X) \text{ and } f(x) = y, \\ x_0 & \text{, if } y \notin f(X). \end{cases}$$

   Then we have

$$(g \circ f)(x) = g(f(x)) = g(y) = x = 1_X(x),\ \forall x \in X,$$

   i.e., $g \circ f = 1_X$.Conversely if $g \circ f = 1_X$, then for x,$x' \in X$ with $f(x) = f(x')$ we have

$$x = 1_X(x) = (g \circ f)(x) = g(f(x)) = g(f(x')) = (g \circ f)(x') = 1_X(x') = x',$$

   i.e., f is injective.

2. Let V be the space of all polynimials. Suppose V is spanned by a finite set of polynomials, $S = \{f_1, f_2, \ldots, f_n\}$. Let f_{k_0} be a polynomial in S such that $\deg f_{k_0} = M \geq \deg f_i$ for all $f_i \in S$. Then all the elements of $L(S)$ are polynomials of degree equal to or less then M. Choose a polynomial g in V such that $\deg g = M + 1$, then $g \notin L(S)$. Thus V is not spanned by S.

3. See solutions of Homework#3.