EE203001 Linear Algebra
Quiz #10 05/06/2003

Quiz Problems:

1. (a) (5 pts.)Prove that a 2×2 matrix A commutes with every 2×2 matrix if and only if A commutes with each of the four matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

   (b) (5 pts.)Find all such matrices A.

2. Prove each of the following statements about $n \times n$ matrices, or exhibit a counter example.

   (a) (2 pts.)If $(A-B)^2 = (A+B)^2$, then $A^2B = BA^2$.

   (b) (2 pts.)If $A^2 = I$, then either $A = I$ or $A = -I$.

   (c) (2 pts.)If $AB + BA = O$, then $A^2B^3 = B^3A^2$.

   (d) (2 pts.)If A and B are nonsigular, then $A + B$ is nonsigular.

   (e) (2 pts.)If A and B are nonsigular, then AB is nonsigular.

   (f) (2 pts.)If the product AB is nonsigular, then both A and B are nonsigular.

   (g) (2 pts.)If A, B, and $A + B$ are nonsigular, then $A - B$ is nonsigular.

   (h) (2 pts.)If $A^2 = O$, then $A = O$.

   (i) (2 pts.)If $A^2 = O$, then $A + I$ is nonsigular.

   (j) (2 pts.)If $(A + 2I)^2 = O$, then A is nonsigular.

3. Determine the inverse of each of the following matrices.

   (a) (5 pts.) $\begin{bmatrix} 0 & 5 & 8 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$.

   (b) (5 pts.) $\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 4 \end{bmatrix}$.