Genetic Algorithm Approach to Fixed-Order Mixed H_2/H_∞ Optimal Deconvolution Filter Designs

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***Abstract*—The mixed H_2/H_∞ optimal deconvolution filter is proposed to achieve the H_2 optimal reconstruction and a desired robustness against the effect of uncertainties in signal processing from the H_∞ norm perspective. However, the conventional mixed H_2/H_∞ optimal design filters are very complicate and are not practical for industrial applications. For simplicity of implementation and conservation of operation time, the fixed-order mixed H_2/H_∞ optimal deconvolution filter design is interesting for engineers in signal processing from the practical design perspective. In this study, to avoid the trap of local minima, a design method based on the genetic algorithm is introduced to treat the nonlinear optimization design problem of the fixed-order mixed H_2/H_∞ deconvolution filter. The convergence property of our design algorithm is also discussed. Finally, an example is presented to illustrate the design procedure and confirm the robustness performance of the proposed method.**

***Index Terms*—Deconvolution, genetic, mixed H_2/H_∞ filter.**

I. Introduction

The deconvolution problem is to reconstruct a signal embedded in noise and to eliminate the effect of any distortion in the signal transmission channel. The deconvolution problem is widely explored in the engineering literature, especially in many signal processing applications [1]–[6], including seismology, equalization, image restoration, etc.

Deconvolution problems are generally solved via the H_2 optimal method by a Wiener filtering technique from the frequency domain perspective [6]–[8] or by a Kalman filtering technique from the time domain perspective [1]–[3]. With an exact knowledge of the channel and suitable assumptions on noise variance, the H_2 filter has some desired optimal properties. However, its performance will deteriorate if the assumptions of the statistics of noise are violated, and the coefficients of the channel are perturbed. Recently, to remedy the shortcomings of the H_2 filter, a robust deconvolution filter design based on H_∞ theory has received great attention for its robustness properties against system uncertainties [8]–[13]. However, the H_∞ robustness design cannot achieve optimal performance. In practical designs, due to perturbation in the system parameters, linearization, and modeling error, the robust mixed H_2/H_∞ optimal control and filter designs have received a great deal of attention from the viewpoint of taking advantage of both H_2 optimal design and H_∞ robustness design [14]–[16]. The proposed mixed H_2/H_∞ optimal deconvolution filter design minimizes an H_2 reconstruction performance index of the nominal system subject to a robustness requirement based on the H_∞ norm to attenuate the performance degradation due to the system's uncertainty. This problem can be interpreted as a problem of optimal reconstruction filter design subject to a robustness constraint against the deterioration due to parameter variation of channel and noise uncertainty. Therefore, it is an important design algorithm from the point of view of practical signal processing.

In general, it is not easy to design a mixed H_2/H_∞ deconvolution filter for a signal transmission system. Furthermore, if the orders of the signal models and the channel dynamics are higher, the difficulty and the complexity on the design of the mixed H_2/H_∞ optimal deconvolution filters increase, resulting in an increase in the cost of realization, and the operation time of the optimal deconvolution filters becomes longer. In practical applications, the fixed-order mixed H_2/H_∞ optimal deconvolution filter designs are interesting due to simplicity in implementation and saving of operation time. Consequently, the design of a mixed H_2/H_∞ deconvolution filter with a desired order is an important topic for practical applications. However, the way to design a fixed-order optimal deconvolution filter to simultaneously achieve H_2 optimal reconstruction and the H_∞ robustness requirement is still a new topic worth investigating.

Since fixed-order deconvolution filter is used, conventional optimization techniques cannot be employed to obtain a closed-form solution of the mixed H_2/H_∞ optimal deconvolution filter. The way to specify the coefficients of the fixed-order deconvolution filter to achieve mixed H_2/H_∞ deconvolution performance is a highly nonlinear constrained optimization problem in which many local minima may exist, and a local minimum may be easily reached via conventional algorithms.

In this study, the design procedure of a fixed-order mixed H_2/H_∞ deconvolution filter is divided into two steps. In the first step, based on Jury's stability criterion, the stability domain of the coefficients of the denominator in a fixed-order deconvolution filter is specified. In the second step, the mixed H_2/H_∞ optimal solution in the stability domain of the coefficients (parameter space) will be searched via the genetic algorithm. Therefore, the stability domain specification can not only guarantee the stability of the deconvolution filter but can also reduce the scope of the search space to improve the convergent rate of the genetic algorithm.

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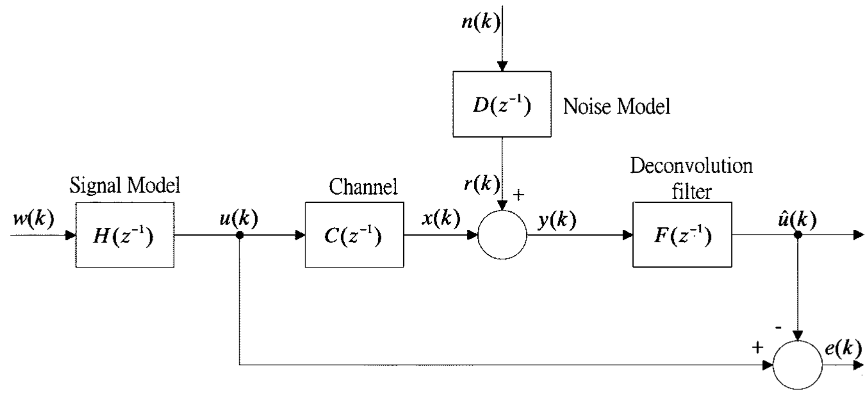

Fig. 1. Linear discrete-time deconvolution system.

Genetic algorithms are parallel, global optimal search techniques that copy natural genetic operations because they simultaneously evaluate many points in the parameter space, and they are more likely to converge toward the global solution [21], [29]. A genetic algorithm does not need to assume that the search space is differentiable or continuous and can also iterate several times on each data received. Genetic algorithms apply operations inspired by the mechanics of natural selection to a population of binary strings encoding the parameter space. At each generation, the genetic algorithm explores different areas of the parameter space and then directs the search to regions where it is of high probability that the mixed H_2/H_∞ performance will be improved. By working with a population of solutions, the algorithms can, in effect, search for many local minima and thereby increase the likelihood of finding the global minimum. Global optimization can be achieved via a number of genetic operators, e.g., selection, mutation, and crossover.

Based on the genetic algorithms, the coefficients (parameters) of the fixed-order deconvolution filter will be tuned in the stability domain to achieve the mixed H_2/H_∞ optimal design. Genetic algorithms are more suitable to solve the iterative fixed-order mixed H_2/H_∞ optimization problem than other major searching methods such as the gradient-based algorithm and the random searching algorithm for the following reasons. First, the search space is large, and there are several minima. Second, the performance surface does not need to be differentiable with respect to the change of parameters. Furthermore, the gradient-based searching algorithm that depends on the existence of derivatives is inefficient and can be caught in a trap of local minima. Third, the likely fit terms are less likely to be destroyed under the genetic operator, thereby often leading to faster convergence. By employing the genetic algorithm to treat our mixed H_2/H_∞ optimization problem, the filter coefficients are first coded to a binary string-cells chromosome. In each generation, these basic genetic operators (i.e., selection, crossover, and mutation) generate a new population with constant population size via the values of the appropriate selections. Finally, the convergence the proposed genetic-based H_2/H_∞ filter design algorithm is proved from the viewpoint of the Markov chain. It is also proved that the proposed method converges to the global mixed H_2/H_∞ optimal solution. A simulation example is given to illustrate the design procedure of the proposed design method and to confirm the robustness performance of the fixed-order mixed H_2/H_∞ deconvolution filter.

II. Problem Description

Consider a discrete deconvolution system as shown in Fig. 1, where the signal $u(k)$ is generated by $H(z^{-1})$ driven by white noise $w(k)$. The signal $u(k)$ is transmitted through a channel system whose transform function is $C(z^{-1})$ and is corrupted by additive colored noise $r(k)$, which is generated by $D(z^{-1})$ and driven by white noise $n(k)$. The received signal $y(k)$ is given by

$$y(k) = x(k) + r(k) = C(z^{-1})u(k) + D(z^{-1})n(k) \tag{1}$$

where z^{-1} is the inverse of z (i.e., unit delay), $w(k)$ and $n(k)$ are uncorrelated and assumed to be zero-mean uncorrelated white Gaussian noises with covariances

$$E[w^2(k)] = \sigma_w^2, \quad E[n^2(k)] = \sigma_n^2, \quad E[w(k) \cdot n(k)] = 0$$

respectively, where σ_w^2 and σ_n^2 are assumed to be positive scales, and the system is assumed to have reached a statistical steady state.

Let $\hat{u}(k)$ denote the estimate of $u(k)$ in the presence of both signal distortion through channel $C(z^{-1})$ and the corrupted noise $r(k)$. The objective of this paper is to find a fixed-order IIR deconvolution filter

$$F(z^{-1}) = \frac{h_0 + h_1 z^{-1} + \cdots + h_i z^{-i} + \cdots + h_l z^{-l}}{1 + f_1 z^{-1} + \cdots + f_i z^{-i} + \cdots + f_l z^{-l}} \tag{2}$$

so that the mixed H_2/H_∞ optimal signal reconstruction is achieved with a prescribed filter order l by the designer. The stability of the fixed-order deconvolution filter $F(z^{-1})$ in (2) is the most important thing. By Jury's stability test [27], the stability domain S of the coefficients $(f_1, \dots f_l)$ can be easily specified. In order to guarantee the stability of $F(z^{-1})$, the following constraint is imposed:

$$(f_1 \cdots f_l) \in S. \tag{3}$$

Remark 1: In FIR case, set $f_1 = f_2 = \cdots = f_l = 0$, and neglect the stability condition (3). $\square$

From Fig. 1, the estimation error is given by

$$\begin{aligned} e(k) &= u(k) - \hat{u}(k) \\ &= [1 - F(z^{-1})C(z^{-1})]H(z^{-1})w(k) \\ &\quad + F(z^{-1})D(z^{-1})n(k). \end{aligned} \tag{4}$$

The power spectral density of $e(k)$ $[\Phi(z^{-1})]$ is given by

$$\begin{aligned} \Phi(z^{-1}) &= (1 - F(z^{-1})C(z^{-1}))H(z^{-1})\sigma_w^2 H^*(z^{-1}) \\ &\quad \times (1 - F(z^{-1})C(z^{-1}))^* \\ &\quad + F(z^{-1})D(z^{-1})\sigma_n^2 D^*(z^{-1})F^*(z^{-1}) \end{aligned} \tag{5}$$

where $H^*(z^{-1})$ denotes the complex conjugate of $H(z^{-1})$, i.e., $H^*(z^{-1}) = H(z)$.

In the fixed-order H_2 optimal deconvolution filter design case, a stable filter $F(z^{-1})$ in (2) will be specified to minimize the following least mean square error [26]:

$$\min_{F(z^{-1})} E\{e^2(k)\} = \min_{F(z^{-1})} \frac{1}{2\pi i} \oint_{|z|=1} \Phi(z^{-1}) \frac{dz}{z} \tag{6}$$

where $\Phi(z^{-1})$ is defined in (5).

In general, the calculus variation technique with the aid of spectral factorization and Cauchy theorem can be employed to solve the above H_2 optimization design problem to obtain a full-order H_2 optimal deconvolution filter [6], [7]. However, in the fixed-order H_2 optimal deconvolution filter design case, a closed-form solution is impossible, and iterative searching methods are necessary to treat this problem.

In practical deconvolution systems, the noise variances σ_w^2 and σ_n^2 may vary, and the coefficients of transmission channel $C(z^{-1})$ may experience perturbation. The manner in which to eliminate the performance degradation due to noise uncertainties and channel perturbation in order to guarantee the reconstruction performance is an important topic in the practical signal deconvolution problem. The H_∞ robustness design can guarantee that the worst-case effect of these noise uncertainties or channel perturbation on the reconstruction performance will be less than a prescribed level [10]–[13], i.e., if the H_∞ norm of error spectrum $\Phi(z^{-1})$ is less than ε, the sensitivity of the reconstruction error $e(k)$ to the noise uncertainties or channel perturbation must be less than ε from the energy point of view. In this study, in order to take advantage of both H_2 optimal reconstruction and less sensitivity to noise uncertainties and channel perturbation, a fixed-order deconvolution filter $F(z^{-1})$ in (2) is specified to achieve the H_2 minimization of reconstruction error in (6) and, at the same time, to satisfy the following H_∞ robustness requirement:

$$\|\Phi(z^{-1})\|_\infty =: \sup_{w\in[0,\pi]} |\Phi(e^{-jw})| < \varepsilon \tag{7}$$

where ε is a positive value, i.e., the optimal reconstruction in (6) and H_∞ robustness in (7) should be satisfied simultaneously.

Remark 2: From (5) and (7), since the entire power spectrum is overridden below a small ε, the effect of noise uncertainties and channel perturbation can be efficiently attenuated. This is the essence of H_∞ robustness design. Since $\Phi(e^{-jw})$ is positive and symmetric within $[-\pi, 0]$ and $[0, \pi]$, we only consider the global peak within $[0, \pi]$ in (7). □

Some techniques, such as inner-outer factorization, Nehari's theorem, and the Riccati equation, have been utilized to solve the above H_∞ robustness problem in (7) to obtain a full-order H_∞ optimal solution [10]–[12]. However, in the fixed-order mixed H_2/H_∞ deconvolution filter case, the conventional techniques cannot be applied to solve this mixed H_2/H_∞ deconvolution filter problem. In this paper, genetic algorithms are employed to treat this design problem.

Genetic algorithms are optimization and machine learning algorithms, initially inspired from the processes of natural selection and evolution genetics. Therefore, in this study, genetic algorithms will be employed to specify the coefficients of fixed-order filter $F(z^{-1})$ in (2) to solve the H_2 optimal deconvolution filter design problem in (6) under the constraint of the H_∞ robustness in (7).

III. Fixed-Order Mixed H_2/H_∞ Optimal Deconvolution Filter Designs

In this section, fixed-order mixed H_2/H_∞ optimal deconvolution filter designs will be introduced

A. Fixed-Order H_2 Optimal Deconvolution Filter

Consider the fixed-order H_2 optimal deconvolution in (6). Let

$$I_2 = \frac{1}{2\pi i} \oint_{|z|=1} \Phi(z^{-1}) \frac{dz}{z} = \frac{1}{2\pi i} \oint_{|z|=1} \frac{B(z)B(z^{-1})dz}{A(z)A(z^{-1})z} \tag{8}$$

where $(B(z)B(z^{-1}))/(A(z)A(z^{-1}))$ denotes the spectral factorization of $\Phi(z^{-1})$ with all the roots of polynomials $A(z)$ and $B(z)$ in $|z| < 1$. In addition, let us perform the following decomposition:

$$\frac{1}{z}\Phi(z^{-1}) = \frac{b_0}{z} + \sum_{i=1}^{l} \left(\frac{b_i}{z + a_i} + \frac{c_i}{z^{-1} + a_i} \right) \tag{9}$$

where a_i, b_i, and c_i are all the functions of the coefficients h_i for $i = 0, 1, \ldots, l$ and f_i for $i = 1, 2, \ldots, l$. Suppose the poles $z = -a_i$ are inside $|z| = 1$ and the poles $z = -(1/a_i)$ are outside $|z| = 1$ for all $i = 1, 2, \ldots, l$. Substituting (9) into (8), we get

$$\begin{aligned} I_2 &= \frac{1}{2\pi i} \oint_{|z|=1} \left(\frac{b_0}{z} + \sum_{i=1}^{l} \left(\frac{b_i}{z + a_i} + \frac{c_i}{z^{-1} + a_i} \right) \right) \\ dz &= \sum_{j=0}^{l} b_j \end{aligned} \tag{10}$$

where $b_j, j = 0, 1, \ldots l$ are the functions of the coefficients h_i and f_i in the last equality of (10), and we employ Cauchy's residue theorem to evaluate the integral in (10). In this situation, the H_2 optimal deconvolution problem in (6) becomes the following minimization problem:

$$\min_{f_i, h_i} I_2 = \min_{f_i, h_i} \sum_{j=0}^{l} b_j(f_1, \ldots f_l, h_0, \ldots h_l). \tag{11}$$

In general, I_2 is a very highly nonlinear function of the coefficients h_i for $i=0,1,\ldots,l$ and f_i for $i=1,2,\ldots,l$. There may exist many local minima. We cannot get a closed-form solution to the I_2 minimization problem in (11). Furthermore, it is still very difficult to find the global minimum of I_2 in (11) by the conventional searching methods.

B. H_∞ Robustness Constraint on Fixed-Order Deconvolution Filter

Consider the fixed-order H_∞ deconvolution design problem in (7). Let

$$\begin{aligned} I_\infty(f_1\ldots f_l, h_0\ldots h_l) &= \|\Phi(z^{-1})\|_\infty \\ &= \sup_{w\in[0,\pi]} \Phi(e^{-jw}) < \varepsilon \end{aligned} \tag{12}$$

where ε is given beforehand. Since we only need to find the global peak of $\Phi(z^{-1})$ within the internal $[0,\pi]$ of the frequency axis in (12), $\|\Phi(z^{-1})\|_\infty$ will be obtained via searching the maximum value of $\Phi(e^{-jw})$ within $w\in[0,\pi]$. To solve the H_∞ robustness problem $\|\Phi(z^{-1})\|_\infty<\varepsilon$ in (12) is to specify the coefficients h_i for $i=0,1,\ldots l$ and f_i for $i=1,2,\ldots l$ so that the maximum value of $\Phi(e^{-jw})$ is less than ε. From the analysis given in the above two subsections, the fixed-order mixed H_2/H_∞ deconvolution problem to be solved is to specify the coefficients $f_1\ldots f_l$ and $h_0\ldots h_l$ of $F(z^{-1})$ to achieve the I_2 minimization in (11) under the robustness constraint $I_\infty<\varepsilon$ in (12). Furthermore, the way to search the coefficients of $F(z^{-1})$ via the genetic algorithm to solve the mixed H_2/H_∞ deconvolution filter design problem is the key point in our design. This nonlinear complicated design problem will be treated by the genetic algorithm in the next section.

IV. Genetic-Based Fixed-Order Mixed H_2/H_∞ Optimal Deconvolution Filter Design

A. Simple Description of Genetic Algorithm

Genetic algorithms belong to the pool of artificial evolution methods. Their initial inspiration comes from the processes of natural selection (Darwinism) and evolutionary genetics. The mathematical framework was developed in the late 1960s and has been presented in Holland's pioneering book [17]. Genetic algorithms have been proven to be efficient in many areas [18]–[25], [28], such as the N-P hard combinational optimization problem [20], system identification [21], [22], images [23], and fuzzy systems [25]. More details about GAs can be found in Goldberg [18].

Genetic algorithms work with a population of binary strings and not the parameters themselves. The coding that has shown to be the optimal one is binary coding. Therefore, the parameter (coefficient) set of the fixed-order deconvolution filter $f_1,\ldots,f_l,h_0\ldots,h_l$ could be coded as binary strings of $0's$ and $1's$. The binary strings called chromosomes then explore the searching space, and each chromosome represents one possible solution to the problem.

Genetic algorithms only require information concerning the quality of the solution produced by each parameter set (i.e., cost function values or evaluation function values). This differs from many optimization methods that require derivative information or, worse yet, complete knowledge of problem structure and parameters. Since genetic algorithms do not require such problem-specific information, they are more suitable than most searching algorithms for the fixed-order mixed H_2/H_∞ deconvolution filter design.

B. Genetic Algorithm for Fixed-Order Mixed H_2/H_∞ Optimal Deconvolution Filter Designs

A simple genetic algorithm for fixed-order mixed H_2/H_∞ optimal deconvolution filter design is composed of three operators:

1) selection;
2) crossover;
3) mutation.

These operators are implemented by performing the basic tasks of coping strings, exchanging portion of strings, and changing the state of a bit from 0 to 1 or 1 to 0. These operators ensure that the best members of the population in parameter space will survive, and their information contents are preserved and combined to generate better offsprings, that is, to improve the performance of the next generation. It is shown in the Schema theorem [18] that the genetic search algorithm converges exponentially from the view point of schema. We describe the genetic algorithms as follows.

1) Encoding: Genetic algorithms work with a population of strings and not the parameter themselves. For simplicity and convenience, binary coding is used in this paper. Based on the binary coding method, the ith element of the coefficient vector $(f_1\ldots,f_l,h_0\ldots h_l)$ is coded as a string of length B_i, which consists of 0 and 1, for the desired resolution R_i. In general, we have

$$R_i=\frac{U_i-L_i}{2^{B_i}-1},\quad \text{for } i=1,\ldots,2l+1$$

where U_i and L_i are the upper and lower bounds of the ith element of the coefficient vector $(f_1\ldots f_l,h_0\ldots h_l)$. In this study, we let $B_i=B$ for all i.

2) Fitness: By the genetic algorithm, the cost function of the mixed H_2/H_∞ filter is defined as

$$\begin{aligned} &\operatorname*{Min}_{f_i,h_i} I_2(f_1\ldots f_l,h_0\ldots h_l) \\ &\text{subject to } I_\infty(f_1\ldots f_l,h_0\ldots h_l)<\varepsilon. \end{aligned} \tag{13}$$

Our design solves the constrained minimization in (13) from the coefficient vectors $(f_1\ldots f_l,h_0\ldots h_l)\in S$, that is, our objective is to search $(f_1\ldots f_l,h_0\ldots h_l)$ with $(f_1\ldots f_l)\in S$ to satisfy the constraint $I_\infty(f_1\ldots f_l,h_0\ldots h_l)<\varepsilon$ and then to achieve the minimization of (11).

In genetic-based design procedure, a chromosome (a possible $(f_1\ldots f_l,h_0\ldots h_l)$) generates a cost function $I_2(f_1\ldots f_l,h_0\ldots h_l)$ and returns a value. The value of the cost is then mapped into a fitness value $E(f_1\ldots f_l,h_0\ldots h_l)$ to fit into the genetic algorithm. The fitness value is a reward based on the performance of the possible solution represented by the string, or it can be thought of as how well a fixed-order deconvolution filter can be tuned according to the string to actually minimize the cost function $I_2(f_1 \ldots f_l, h_0 \ldots h_l)$. The better the solution encoded by a string (chromosome), the higher the fitness. To minimize $I_2(f_1 \ldots f_l, h_0 \ldots h_l)$ is equivalent to getting a maximum fitness value in the genetic searching process; a chromosome that has a lower $I_2(f_1 \ldots f_l, h_0 \ldots h_l)$ should be assigned a large fitness value. Then, the genetic algorithm tries to generate better offspring to improve the fitness. Therefore, a better deconvolution filter could be obtained by better fitness in genetic algorithms. Therefore, we let

$$E(f_1 \ldots f_l, h_0 \ldots h_l) \propto \frac{1}{I_2(f_1 \ldots f_l, h_0 \ldots h_l)}. \quad (14)$$

There are a number of methods to perform this mapping, which are known as fitness techniques. In this paper, we use so-called windowing [24], as described in Fig. 2. By using the windowing method as a linear straightforward method of transforming cost values into fitness, we can avoid the case of having a smaller fitness difference between good and bad chromosomes, which will decrease the performance of the genetic algorithm. In Fig. 2, $I_{2_{\mathrm{worst}}}$ is the largest cost value in the generation being evaluated, and $I_{2_{\mathrm{best}}}$ is the smallest cost value. E_w and E_b are the corresponding fitness values. Furthermore, the relation between $E(f_1 \ldots f_l, h_0 \ldots h_l)$ and $I_2(f_1 \ldots f_l, h_0 \ldots h_l)$ could be expressed as a linear equation

$$E(f_1 \ldots f_l, h_0 \ldots h_l) = mI_2(f_1 \ldots f_l, h_0 \ldots h_l) + q \quad (15)$$

where the constant m and q are computed by $E_b, E_w, I_{2_{\mathrm{best}}}$ and $I_{2_{\mathrm{worst}}}$ in each generation as

$$m = \frac{E_b - E_w}{I_{2_{\mathrm{best}}} - I_{2_{\mathrm{worst}}}}, \quad q = E_b - I_{2_{\mathrm{best}}} \frac{E_b - E_w}{I_{2_{\mathrm{best}}} - I_{2_{\mathrm{worst}}}}. \quad (16)$$

The following three operations are employed in the genetic algorithm to search for the global optimal solution (i.e., the best fitness) in (13) without becoming trapped at local minima [22]–[24].

3) Selection: Selection is based on the principle of survival of the fitness. The fitness of the ith string E_i is assigned to each individual string in the population where higher E_i means better fitness. These strings with large fitness would have large number of copies in the new generation. For example, the ith string with high fitness values E_i is given a proportionately high probability of selection pr_i according to the distribution

$$\mathrm{pr}_i = \frac{E_i}{\sum E_i}. \quad (17)$$

Once the strings are selected or copied for possible use in the next generation, they are selected in a mating pool, where they await the action of the other two operators: crossover and mutation.

4) Crossover: If the chromosomes are operated only by selection, they search toward the best existing individuals but do not create any new individuals. By the second operator (i.e., crossover), strings exchange information via probabilistic decision. Crossover provides a mechanism for strings to mix and match their desirable qualities through a random process.

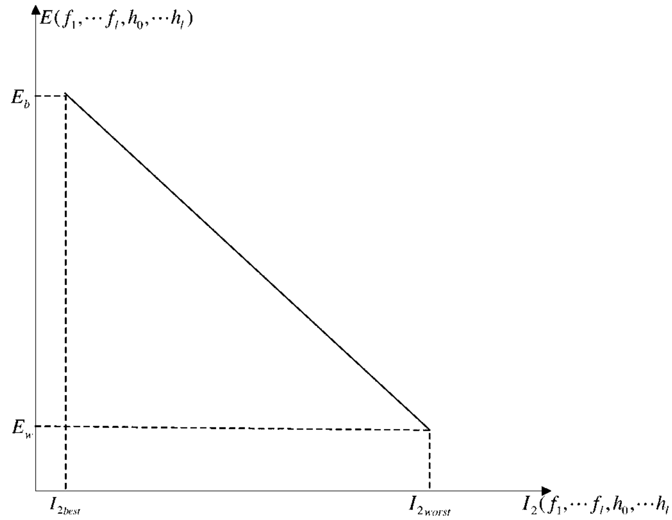

Fig. 2. Relation between $I_2(f_1, \ldots f_l, h_0, \ldots h_l)$ and $E(f_1, \ldots f_l, h_0, \ldots h_l)$.

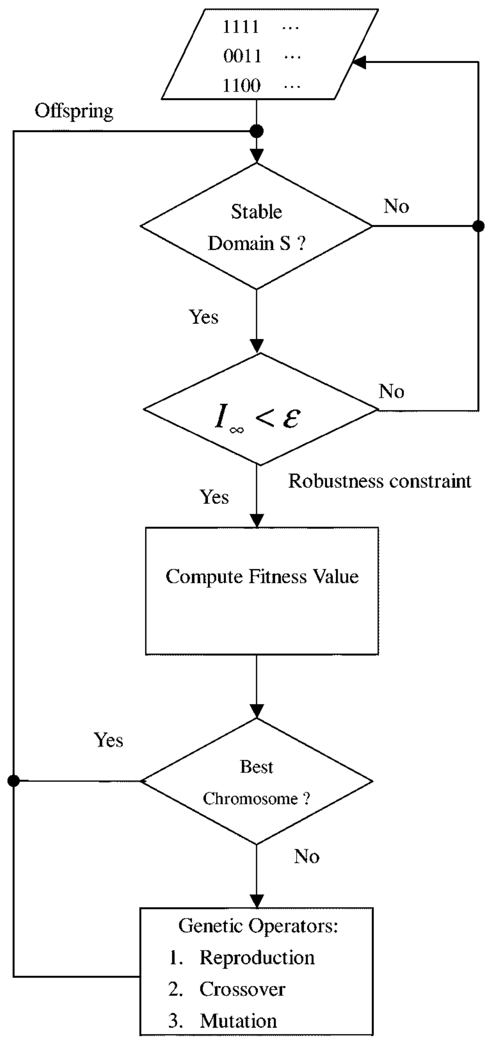

Fig. 3. Design procedure.

After selection, simple crossover proceeds in three steps. First, two newly reproduced strings are selected from the mating

TABLE I
PERFORMANCES FOR VARIOUS CRITERIONS AND VARIOUS FILTER ORDERS

Method	H_2 Criterion by genetic		Mixed H_2/H_∞ Criterion by genetic		H_∞ Criterion by genetic		MMSE in [31]	Mixed H_2/H_∞ Criterion by genetic without elitist	
Filter classification	Second order IIR filter	Third order IIR filter	Second order IIR filter	Third order IIR filter	Second order IIR filter	Third order IIR filter	Fourth order IIR filter	Second order IIR filter	Third order IIR filter
Minimum mean square error by 200 independent MC	0.15	0.1414	0.1618	0.1593	0.262	0.24	0.154	0.1719	0.1687
Deconvolution Filters	$F_1(z^{-1})$	$F_2(z^{-1})$	$F_3^*(z^{-1})$	$F_4^*(z^{-1})$	$F_5(z^{-1})$	$F_6(z^{-1})$	$F_7(z^{-1})$	$F_8(z^{-1})$	$F_9(z^{-1})$

pool produced by selection. Second, a position along the two strings is selected uniformly at random. This is illustrated below, where two binary coded strings $(f_1 \dots f_l, h_0 \dots h_l)_A$ and $(f_1 \dots f_l, h_0 \dots h_l)_B$ are aligned for crossover:

$$\begin{array}{lcccccc} (f_1 \dots f_l, h_0 \dots h_l)_A = 1 & 0 & & \cdots & & 1 & 1 \\ (f_1 \dots f_l, h_0 \dots h_l)_B = 0 & 1 & & \cdots & & 0 & 0 \\ & \uparrow & \text{cross point.} & & & & \end{array}$$

The third step is to exchange all characters following the crossing sit. For example, the two strings $(f_1 \dots f_l, h_0 \dots h_l)_A$ and $(f_1 \dots f_l, h_0 \dots h_l)_B$ with a crossover sitting at 2 become

$$\begin{aligned} (f_1 \dots f_l, h_0 \dots h_l)_{A'} &= 11 \dots 00 \\ (f_1 \dots f_l, h_0 \dots h_l)_{B'} &= 00 \dots 11. \end{aligned}$$

Although crossover uses random choice, it should not be thought of as a random walk through the search space. When combined with selection, it is an effective means of exchanging information and combing portions of high-quality solutions.

5) Mutation: Selection and crossover give genetic algorithms most of their search power. The third operator (mutation) enhances an ability of genetic algorithms to find a near-optimal solution. Mutation is the occasional alternation of a value at a particular string position, which is an insurance policy against the permanent loss of any simple bit, and it is applied with a low probability such that it is chosen so that on an average one string in the population is mutated. For example

$$\begin{array}{lcccc} (f_1 \dots f_l, h_1 \dots h_l) = 1 & 1 & \cdots & 0 & 0 \\ & \downarrow \ \text{mutation} & & & \\ (f_1 \dots f_l, h_1 \dots h_l)' = 1 & 0 & \cdots & 0 & 0. \end{array}$$

In the case of binary coding, the mutation operator simply flips the state of a bit from 1 to 0 or 1 to 0. Mutation should be used sparingly because it is a random search operator, and with high mutation rates, the algorithm should become little more than a random search.

TABLE II
FILTER STYLE

Filter
$F_1(z^{-1}) = \dfrac{0.8789 - 0.0195z^{-1} - 0.0195z^{-2}}{1 + 1.6z^{-1} + 0.0655z^{-2}}$
$F_2(z^{-1}) = \dfrac{0.918 - 0.2148z^{-1} - 0.0195z^{-2} - 0.0195z^{-3}}{1 + 1.4297z^{-1} + 0.2622z^{-2} - 0.1921z^{-3}}$
$F^*{}_3(z^{-1}) = \dfrac{0.8789 - 0.0195z^{-1} - 0.0195z^{-2}}{1 + 1.6185z^{-1} + 0.06717z^{-2}}$
$F^*{}_4(z^{-1}) = \dfrac{0.7617 - 0.3711z^{-1} - 0.0195z^{-2} - 0.0195z^{-3}}{1 + 1.1436z^{-1} - 0.184z^{-2} - 0.3597z^{-3}}$
$F_5(z^{-1}) = \dfrac{0.5859 - 0.0195z^{-1} - 0.0195z^{-1}}{1 + 1.509z^{-1} + 0.562z^{-2}}$
$F_6(z^{-1}) = \dfrac{0.625 - 0.1367z^{-1} - 0.0586z^{-2} - 0.0195z^{-3}}{1 + 1.2469z^{-1} + 0.0964z^{-2} - 0.2086z^{-3}}$
$F_7(z^{-1}) = \dfrac{1.11 + 1.44z^{-1} + 0.453z^{-2} - 0.0837z^{-3} - 0.0462z^{-4}}{1.1 + 3.3z^{-1} + 3.663z^{-2} + 1.776z^{-3} + 0.3168z^{-4}}$
$F_8(z^{-1}) = \dfrac{0.8203 - 0.01953z^{-1} + 0.1172z^{-2}}{1 + 1.3605z^{-1} + 0.4212z^{-2}}$
$F_9(z^{-1}) = \dfrac{0.8984 + 0.5859z^{-1} + 0.0586z^{-2} - 0.0977z^{-3}}{1 + 2.3419z^{-1} + 1.8026z^{-2} + 0.4543z^{-3}}$

Once the best fitness value $E^*(f_1^* \dots f_l^*, h_0^* \dots h_l^*)$ is obtained by the above genetic algorithm by the inverse relationship in (14), the corresponding minimum H_2 performance $I_2^*(f_1^* \dots f_l^*, h_0^* \dots h_l^*)$ is also obtained. Then, we get the corresponding fixed-order mixed H_2/H_∞ deconvolution filter as the following if $I_\infty^*(f_1^* \dots f_l^*, h_0^* \dots h_l^*) < \varepsilon$ in (12) is satisfied:

$$F^*(z^{-1}) = \frac{h_0^* + h_1^* z^{-1} + \cdots + h_i^* z^{-i} + \cdots + h_l^* z^{-l}}{1 + f_1^* z^{-1} + \cdots + f_i^* z^{-i} + \cdots + f_l^* z^{-l}}. \tag{18}$$

TABLE III
THE ROBUSTNESS PROPERTY OF DECONVOLUTION FILTERS UNDER VARIOUS CHANNEL PERTURBATIONS.

Channel perturbations \ Error variance \ Deconvolution filter	Proposed method		MMSE in [31]	H_2
	$F^*_3(z^{-1})$	$F^*_4(z^{-1})$	$F_7(z^{-1})$	$F_2(z^{-1})$
$C(z^{-1})=\frac{0.9+2.17z^{-1}+01.66z^{-2}+0.4z^{-3}}{1+0.6z^{-1}-0.01z^{-2}-0.006z^{-3}}$ (Original)	0.1618	0.1593	0.154	0.1414
$C(z^{-1})=\frac{0.9+2.17z^{-1}+1.76z^{-2}+0.4z^{-3}}{1+0.6z^{-1}-0.01z^{-2}-0.006z^{-3}}$	0.36	0.53	1.49	0.745
$C(z^{-1})=\frac{1+1.8z^{-1}+0.89z^{-2}}{1+0.645z^{-1}-0.0065z^{-2}-0.003z^{-3}}$	0.38	0.57	1.59	0.7927
$C(z^{-1})=\frac{0.9+2.26z^{-1}+1.887z^{-2}+0.566z^{-3}+0.1z^{-4}}{1+0.72z^{-1}+0.02z^{-2}-0.0062z^{-3}-0.001z^{-4}}$	0.27	0.31	0.84	0.455
$C(z^{-1})=\frac{0.9+2.17z^{-1}+1.6z^{-2}+0.35z^{-3}}{1+0.6z^{-1}-0.015z^{-2}-0.016z^{-3}}$	0.21	0.19	0.25	0.19
$C(z^{-1})=\frac{0.9+2.17z^{-1}+1.66z^{-2}+0.34z^{-3}}{1+0.68z^{-1}-0.01z^{-2}-0.006z^{-3}}$	0.28	0.31	0.72	0.41

C. Design Procedure

Based on the above analysis, the design procedure of fixed-order mixed H_2/H_∞ optimal deconvolution filter design is divided into the following steps.

Step 0) Given the order l of the deconvolution filter $F(z^{-1})$, H_∞ robustness constraint ε, the channel model $C(z^{-1})$, signal model $H(z^{-1})$, noise model $D(z^{-1})$, noise covariances σ_w^2 and σ_n^2, and the genetic parameters.

Step 1) Generate random population of N chromosomes.

Step 2) Use Jury's stability test [27] to specify the parameter domain S of the coefficients $(f_1\dots f_l)$ of denominator of $F(z^{-1})$ to guarantee the stability of deconvolution filter $F(z^{-1})$. If the coefficients $(f_1\dots f_l)\notin S$, then renew the chromosomes.

Step 3) Check the H_∞ robustness constraint $I_\infty(f_1\dots f_l,h_0\dots h_l) < \varepsilon$. If the robustness constraint is not satisfied, then renew the chromosomes.

Step 4) Compute the H_2 performance

$$I_2(f_1\dots f_l,h_0\dots h_l)=\sum b_i(f_1\dots f_l,h_0\dots h_l).$$

Step 5) Compute the corresponding fitness value

$$E(f_1\dots f_l,h_0\dots h_l)=mI_2(f_1\dots f_l,h_0\dots h_l)+q.$$

Step 6) Retain the best chromosome intact into the next generation.

Step 7) Use genetic operators (selection, crossover, and mutation) to generate new chromosomes into the next generation.

Then, repeat the procedure from step 2 to step 7 until a suitable parameter(chromosome) set is obtained. The flowchart of the design procedure for the proposed mixed H_2/H_∞ deconvolution filter is presented in Fig. 3 for illustration.

Remark 3: In this work, we use elitist strategy, when the rightful chromosomes in mating pool are not large enough, they can cause genetic algorithm premature convergence to local optima; therefore, irregular chromosomes in step 2 and step 3 must be updated by the new chromosomes. □

D. Convergence Analysis

A genetic algorithm with elitist strategy is chosen in this paper to guarantee the convergence of the parameter estimation algorithm. The elitist strategy always survives the best chromosome intact into next generation. The convergence of genetic algorithm with elitist strategy has been proved in [29] from the viewpoint of the Markov chain. It has also been proved that the genetic algorithm converges to the global optimum as the generation approaches to infinity. In this paper, the convergence analysis of the proposed parameter searching algorithm for a fixed-order mixed H_2/H_∞ deconvolution filter is discussed following the path of proof of genetic algorithm in [29].

Let all of the chromosomes in a generation be arranged into a binary string with length $N\cdot L_t$ (where N is the population size, L_t is the length of each chromosome, and $L_t=(2l+1)*B$). The size of the state space of the Markov chain is denoted by $O=2^{N\cdot L_t}$. We denote the $\Omega^t=\{(f_1\dots f_l,h_0\dots h_l)_1^t,(f_1\dots f_l,h_0\dots h_l)_2^t,\dots,(f_1\dots f_l,h_0\dots h_l)_N^t\}$ in which $(f_1\dots f_l,h_0\dots h_l)_k^t$ denotes the kth chromosome in a population at generation t. A simple genetic algorithm uses three operations, i.e., selection, crossover, and mutation, to change state from the state of the operations of the population at generation t, say, state j, to another state at generation $t+1$, say, state k. The evolution of a simple genetic algorithm is regarded as a finite ergodic Markov chain [30]. The probabilistic changes of the genes within the population caused by genetic operations are captured by the transition matrix T of a Markov chain with dimension $NL_t \times NL_t$ in which the matrix $T = R_e \cdot C_r \cdot M_u$ (R_e, C_r and M_u denote the intermediate transitions caused by selection, crossover, and mutation, respectively).

The fitness $E((f_1 \ldots f_l, h_0 \ldots h_l)_k^t)$ of the kth chromosome at generation t is a positive function. We wish to find the global maximum E^* of $E(f_1 \ldots f_l, h_0 \ldots h_l)_k^t$. Let us denote the best chromosome at generation t as

$$\widehat{\Omega^t} = \Big\{(f_1 \ldots f_l, h_0 \ldots h_l)_k^t : 1 \le k \le N \\ \mid E((f_1 \ldots f_l, h_0 \ldots h_l)_k^t) \\ = \max_{1 \le k \le N} I_2((f_1 \ldots f_l, h_0 \ldots h_l)_k^t)\Big\}.$$

Next, we will show that the transition matrix T of a simple genetic algorithm is positive (i.e., every element $T_{i,j} > 0$ for $i,j = 1, 2, \ldots, N \cdot L_t$).

The intermediate transition matrices of T (i.e., R_e, C_r, and M_u) are stochastic because each of them is a state transition matrix, and each state of O is translated probabilistic to another state of O. The transition state j changes to state k by mutation, whose probability can be written as $m_{j,k} = p_m^{H_{j,k}}(1-p_m)^{N \cdot L_t - H_{j,k}} > 0$ for all $j, k \in S$, where p_m is the mutation probability, and $H_{j,k}$ is the Hamming distance [29]. Thus, M_u is positive (if $p_m \neq 0$). Next, let $U = C_r \cdot M_u$, and let $T = R_e \cdot U$. Since C_r is a stochastic transition matrix, there exists at least one positive element in each row of C_r; then, $U_{i,j} = \sum_{k=1}^{N \cdot L_t} C_{r_{i,k}} \cdot M_{u_{k,j}} > 0$ for all $i, j \in 1, 2, \ldots, N.L_t$, i.e., U_{ij} is positive. Similarly, R_e is stochastic, and therefore, T is positive.

After the positiveness of the transition matrix T is shown, the convergence of the genetic parameter search is discussed as follows. In an ergodic Markov chain, the expected transition time between initial state j and any other state k is finite [30]. Therefore, a simple genetic algorithm can reach the best fitness E^*(i.e., the global maximum fitness value) in finite generations. However, the best fitness may not survive in the next generation because in a simple genetic algorithm, the best chromosome is not reserved in each generation. In this paper, in order to avoid the drawback of the simple genetic algorithm, the best chromosome will remain intact in each generation, i.e., the genetic algorithm contains the elitist strategy. In this situation, if we assume that E^* is found at generation t, the $\widehat{\Omega^i} = E^*$ for all $i \ge t$. Therefore, we can prove

$$\lim_{t \to \infty} E(\hat{\Omega}^t) = E^*.$$

Actually, the property where the proposed algorithm can converge to the global optimum as time approaches infinity is not of practical interest because the algorithm is generally performed over finite generations.

TABLE IV
ROBUSTNESS OF DECONVOLUTION FILTERS AGAINST VARIOUS NOISE UNCERTAINTIES

Deconvolution filter / Error variance / The uncertainties of noises	Proposed method $F^*_3(z^{-1})$	Proposed method $F^*_4(z^{-1})$	MMSE in [31] $F_7(z^{-1})$
$\sigma_w^2 = 1, \sigma_n^2 = 1$ (SNR = 0 db)	0.8788	0.689	1.1
$\sigma_w^2 = 1, \sigma_n^2 = 0.5012$ (SNR = 3 db)	0.483	0.4022	0.58
$\sigma_w^2 = 1, \sigma_n^2 = 0.3162$ (SNR = 5 db)	0.33	0.29	0.3865
$\sigma_w^2 = 1, \sigma_n^2 = 0.1995$ (SNR = 7 db)	0.2385	0.22	0.269
$\sigma_w^2 = 1, \sigma_n^2 = 0.1$ (SNR = 10 db)	0.1618	0.1593	0.154

TABLE V
ROBUSTNESS DECONVOLUTION FILTERS UNDER THE PERTURBATIONS OF NOISE CHANNEL

Deconvolution filter / Error variance / The perturbation of noise channel	Proposed method $F^*_3(z^{-1})$	Proposed method $F^*_4(z^{-1})$	MMSE in [31] $F_7(z^{-1})$
$D(z^{-1}) = \frac{1+1.67z^{-1}+0.66z^{-2}}{1-0.01z^{-2}}$	0.181	0.188	0.23
$D(z^{-1}) = \frac{1+1.9z^{-1}+0.72z^{-2}}{1+0.01z^{-1}-0.012z^{-2}}$	0.2844	0.3	0.797
$D(z^{-1}) = \frac{1+1.75z^{-1}+0.82z^{-2}}{1-0.015z^{-2}}$	0.166	0.18	0.22
$D(z^{-1}) = \frac{1+1.65z^{-1}+0.62z^{-2}}{1-0.009z^{-2}}$	0.1727	0.178	0.195

V. DESIGN EXAMPLE

A numerical example is given to demonstrate the feasibility of applying genetic algorithm to the design of fixed-order mixed H_2/H_∞ optimal deconvolution filter. In the following example, the parameters of genetic algorithm are set as follows [19], [21], [29]:

$$N = 200, \quad p_m = 0.01, \quad B = 15 \text{ bit}.$$

where

- N number of population;
- p_m mutation probability;
- B length of every parameter.

Consider a nonminimum phase deconvolution system in Fig. 1 with the received signal corrupted by colored noise such that

$$y(k) = C(z^{-1})u(k) + D(z^{-1})n(k), u(k) = H(z^{-1})w(k)$$

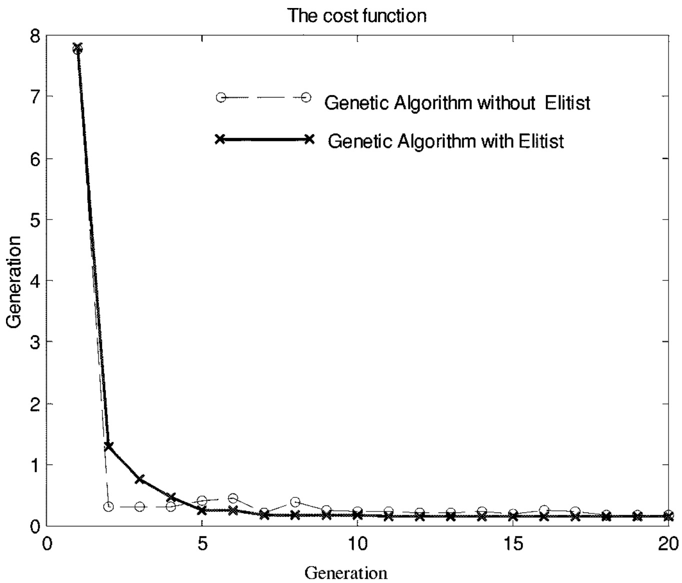

Fig. 4. Cost functions of the second order mixed H_2/H_∞ deconvolution filters $F_3(z^{-1})$ and $F_8(z^{-1})$ via genetic algorithm.

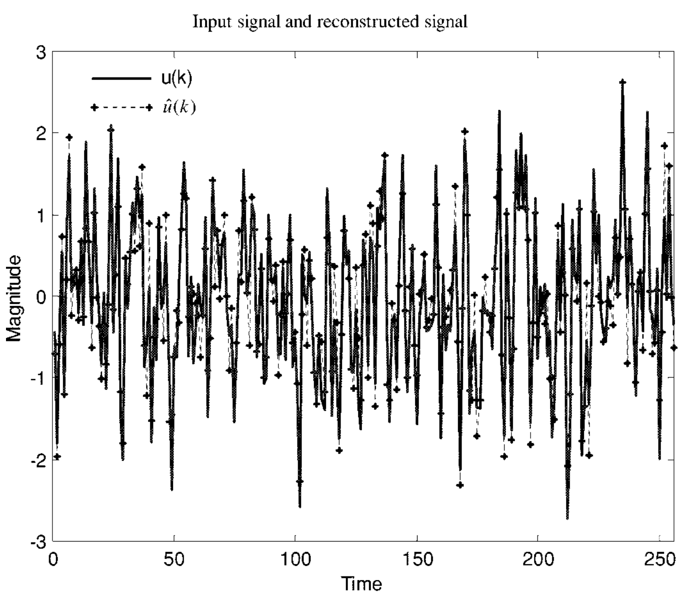

Fig. 5. Input signal $u(k)$ and the reconstructed signal $\hat{u}(k)$ by the second order H_2/H_∞ deconvolution filter $F_3(z^{-1})$.

where the robustness constraint, signal, channel, and noise models are described as follows:

$$\varepsilon = 0.8, H(z^{-1}) = \frac{1+0.8z^{-1}}{1+0.5z^{-1}},$$
$$C(z^{-1}) = \frac{0.9+2.17z^{-1}+1.66z^{-2}+0.4z^{-3}}{1+0.6z^{-1}-0.01z^{-2}-0.006z^{-3}}$$
$$D(z^{-1}) = \frac{(1+0.9z^{-1})(1+0.8z^{-1})}{(1+0.1z^{-1})(1-0.1z^{-1})}.$$

The driving signal $\{w(k)\}$ and disturbance noise $\{n(k)\}$ are also assumed to be independent, stationary, and white with zero mean and variances as given by $\sigma_w^2 = 1, \sigma_n^2 = 0.1$, respectively. The error variance of various criterion are tabulated in

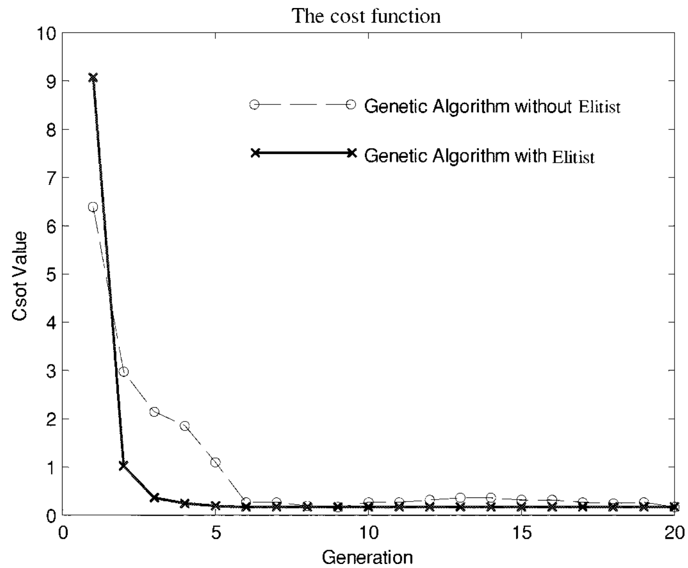

Fig. 6. Cost functions of the third-order mixed H_2/H_∞ deconvolution filters $F_4(z^{-1})$ and $F_9(z^{-1})$ via the genetic alogrithm.

Table I. Inspecting Tables I, III, IV, and V, the performance of mixed H_2/H_∞ approaches the H_2 criterion in the case without channel perturbation, and the robustness is more than that of the minimum mean square error (MMSE) deconvolution filter, while channel suffers from small perturbation or noise uncertainties; therefore, the proposed fixed-order mixed H_2/H_∞ is more appealing for practical applications. By the proposed design algorithms, the second-order mixed H_2/H_∞ optimal deconvolution filters $F_3^*(z^{-1})$ and the third-order mixed H_2/H_∞ optimal deconvolution filters $F_4^*(z^{-1})$ are given in Table II. The convergence of cost function $I_2(f_1, f_2, h_0, h_1, h_2)$ of the second-order mixed H_2/H_∞ optimal filter via the genetic algorithm is shown in Fig. 4. The cost function converges exponentially. The input signal $u(k)$ and the reconstructed signal $\hat{u}(k)$ of the second-order mixed H_2/H_∞ deconvolution filter $F_3^*(z^{-1})$ are shown in Fig. 5. The convergence of cost function $I_2(f_1, f_2, f_3, h_0, h_1, h_2, h_3)$ of the third-order mixed H_2/H_∞ optimal deconvolution filters $F_4^*(z^{-1})$ in the genetic algorithm process is shown in Fig. 6. It can be seen that it converges exponentially. The input signal $u(k)$ and the reconstructed signal $\hat{u}(k)$ by $F_4^*(z^{-1})$ are shown in Fig. 7. The results indicate that we can obtain a good reconstruction performance.

In order to illustrate the robustness of the proposed design algorithm, a full-order deconvolution filter obtained by the orthogonal principle in [31], in which the design performances are based on the MMSE criterion, is found for SNR $= 10$ db in Table I. Now, assume that channel parameters, noise channel parameters, and noise variance σ_n^2 have been changed due to the perturbation in the coefficients. The robustness performance of various perturbation cases are given in Tables III–V. All of these results are obtained by 200 independent Monte Carlo (MC) simulation runs for each data length of 256 points. By inspection of Tables III–V, and while the channel's parameters, noise channel's parameters, or noise variance's have some variations, the mixed H_2/H_∞ deconvolution filter of the proposed method is less influenced than the MMSE deconvolution filter in [31]. Thus, we can conclude that the proposed fixed-order

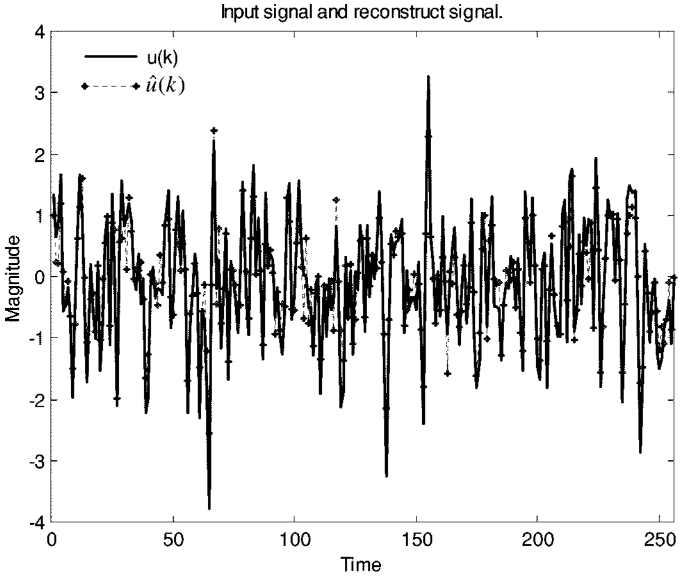

Fig. 7. Input signal $u(k)$ and the reconstructed signal $\hat{u}(k)$ by the second-order H_2/H_∞ deconvolution filter $F_4(z^{-1})$.

mixed H_2/H_∞ deconvolution is more robust than the one based on the MMSE criterion.

VI. Conclusion

In this paper, design methods of the fixed-order mixed H_2/H_∞ optimal deconvolution filters have been introduced via the genetic algorithm. This deconvolution filter design method takes advantage of H_2 optimal reconstruction performance and H_∞ robustness against the channel variation and noise uncertainties. The near mixed H_2/H_∞ optimal design is obtained by genetic operations such as selection, mutation, and crossover. The simulations results indicate that the genetic-based design algorithms converge exponentially, and the reconstruction performance is acceptable even if the order of the deconvolution filter is lower. Furthermore, the proposed fixed-order mixed H_2/H_∞ deconvolution filter possesses more robustness properties than the conventional deconvolution filter under channel perturbations and noise uncertainties. There is a tradeoff between the order of deconvolution filter and the reconstruction performance. The proposed design methods are suitable for lower order optimal deconvolution filter design with the simplicity of implementation as well as saving of operation time and are useful for practical application in the signal reconstruction problems with a high order channel and signal model.

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